Kuwait University	Math 101	Date:	April 19, 2003
Dept. of Math. & Comp. Sci.	First Exam	Duration:	75 minutes

Calculators, mobile phones, pagers and all other mobile communication equipments are not allowed

Answer the following questions:

1. Evaluate the following limits, if exist:

   (a) $\lim_{x\to 1} \dfrac{\sqrt{x}-1}{x^2-1}$. (3 pts.)

   (b) $\lim_{x\to 0} \dfrac{\sin x + x^3 \sin\frac{1}{x}}{x}$. (3 pts.)

2. Find the vertical and horizontal asymptotes, if any, of

$$f(x) = \frac{|x|}{x^2 - x}.$$ (5 pts.)

3. Classify the discontinuities of

$$f(x) = \begin{cases} \dfrac{x-2}{x^3-8} & \text{if, } \; x > 0, \\ \dfrac{\sin[\pi(x+1)]}{(x+1)^2} & \text{if, } \; x \le 0, \end{cases}$$

as removable, jump or infinite. (5 pts.)

4. (a) State the Intermediate Value Theorem. (1 pt.)

   (b) Show that $f(x) = \dfrac{x^3 - x + 1}{x^2 + 1}$ has a horizontal tangent in $[0, 1]$. (2 pts.)

5. (a) If $y = \left(\dfrac{u-1}{u+1}\right)^3$ and $u = \sqrt[3]{2 - 3t - t^2}$, find $\dfrac{dy}{dt}$ at $t = 2$. (3 pts.)

   (b) Find the point at which the graph of $f(x) = 5(x+1)^{2/5} + 2(x+2)^{3/2}$ has a cusp. (3 pts.)

(Good Luck)

Kuwait University	Math 101	Date: April 19, 2003
Dept. of Math. & Comp. Sci.	First Exam	Answers Key

1. (a) $\lim_{x\to 1} \frac{\sqrt{x}-1}{x^2-1} \times \frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+1)(\sqrt{x}+1)} = \boxed{\frac{1}{4}}$.

   (b) For $x \neq 0, -1 \leq \sin\frac{1}{x} \leq 1 \Rightarrow -x^2 \leq x^2 \sin\frac{1}{x} \leq x^2$.
   $\lim_{x\to 0}(-x^2) = 0 = \lim_{x\to 0} x^2$, then from the Sandwich Theorem $\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$.
   $\lim_{x\to 0} \frac{\sin x + x^3 \sin\frac{1}{x}}{x} = \lim_{x\to 0} \frac{\sin x}{x} + \lim_{x\to 0}(x^2 \sin\frac{1}{x}) = 1 + 0 = \boxed{1}$

2. $\lim_{x\to\infty} \frac{|x|}{x(x-1)} = 0 \Rightarrow \boxed{y=0}$ is *H.A* for f & $\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{-x}{x^2-x} = 0 \Rightarrow \boxed{y=0}$ is *H.A* for f.

   $\lim_{x\to 1^{\pm}} f(x) = \lim_{x\to 1^{\pm}} \frac{x}{x(x-1)} = \lim_{x\to 1^{\pm}} \frac{1}{x-1} = \boxed{\pm\infty} \Rightarrow \boxed{x=1}$ is *V.A* for f.

   $\lim_{x\to 0^{\pm}} f(x) = \lim_{x\to 0^{\pm}} \frac{|x|}{x(x-1)} = \boxed{\mp 1}$

3. $\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x-2}{(x-2)(x^2+2x+4)} = \boxed{\frac{1}{12}} \Rightarrow$The graph of f has a *removable* discontinuity at $\underline{x=2}$.

   $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{x-2}{x^3-8} = \boxed{\frac{1}{4}}$ & $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sin[\pi(x+1)]}{(x+1)^2} = \boxed{0} \Rightarrow$The graph of f has a *Jump* discontinuity at $\underline{x=0}$.

   $\lim_{x\to -1} f(x) = \left(\pi \lim_{\pi(x+1)\to 0} \frac{\sin[\pi(x+1)]}{\pi(x+1)}\right) \times \lim_{x\to -1} \frac{1}{(x+1)} = \pi \lim_{x\to -1} \frac{1}{(x+1)}$.

   $\lim_{x\to -1^{\pm}} \frac{1}{(x+1)} = \pm\infty \Rightarrow$The graph of f has an *infinite* discontinuity at $\underline{x=-1}$.

4. (b) $f'(x) = \frac{x^3-x+1}{x^2+1} = \frac{x^4+4x^2-2x-1}{(x^2+1)^2}$. f' is continuous on $[0,1]$ & $f'(1) > 0$, $f'(0) < 0$.
   From the Intermediate Value Theorem, $\exists c \in (0,1)$ such that $f'(c) = 0$. Thus f has a horizontal tangent in $[0,1]$.

5. (a) $u(2) = -2$, $\frac{dy}{du} = 6\frac{(u-1)^2}{(u+1)^4}$, $\Rightarrow \left.\frac{dy}{du}\right|_{u=-2} = \boxed{54}$

   $\frac{du}{dt} = -\frac{2t+3}{3\,(2-3t-t^2)^{2/3}}$, $\Rightarrow \left.\frac{du}{dt}\right|_{t=2} = \boxed{-\frac{7}{12}}$.

   $\left.\frac{dy}{dt}\right|_{\substack{u=-2\\t=2}} = \left.\frac{dy}{du}\right|_{u=-2} \times \left.\frac{du}{dt}\right|_{t=2} = 54 \times \left(-\frac{7}{12}\right) = \boxed{-\frac{63}{2} = -31\frac{1}{2}}$

   (b) $f'(x) = \frac{2}{(x+1)^{3/5}} + 3\,(x+2)^{1/2}$. f is continuous at $x=-1$, $\lim_{x\to -1^{\pm}} f'(x) = \pm\infty \Rightarrow$ The graph of f has a cusp at $\boxed{x=-1}$.